

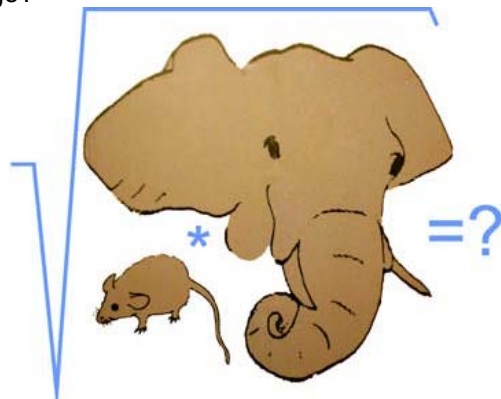
## Questions for a better understanding of the size-dependent ratio of surface/cross-sectional area to volume/mass

Adapted from

Georg Glaeser: Der mathematische Werkzeugkasten (4.Aufl.),  
Springer Spektrum, Heidelberg 2014

**Fundamental idea: If a solid is enlarged by the factor  $k$ , then its surface/cross-sectional area is enlarged by a factor of  $k^2$ , and its volume/mass is enlarged by a factor of  $k^3$ . In this sense, the volume increases  $k$  times faster than surface.**

Question: Why are the African elephant's ears so large?



Short answer:

For better cooling.

Detailed answer with mathematical reasoning:

Large animals have a smaller surface in proportion to their volume.

Cooling occurs through the surface. African elephants inhabit hot climates. Without an enlargement of their ear surface, they would overheat.

Consider: Mammoths lived in cold climates, and thus they had thick fur + small ears.

More questions on the same topic:

- \* Why can't birds and mammals grow smaller than a hummingbird or a shrew mouse?
- \* Why do great whales migrate across thousands of kilometres from polar regions to tropical regions, where food is scarce?
- \* Why do children feel cold stronger than adults?
- \* Why do small birds find it easy to incubate their eggs by sitting on them, and why is the same task so difficult for ostriches?
- \* Why do glaciers melt so slowly, and ice cubes so quickly?
- \* Why does caster sugar dissolve faster than coarse-grained sugar?

Why are ants so strong?

Short answer:

Because they are so small.



Detailed answer with mathematical reasoning:

Muscular strength depends on the muscle's cross-sectional area, which, if the organism is scaled down, decreases at a square ratio. However, mass decreases as at a cubic ratio – thus, it decreases much more quickly. For this reason, the muscles of small organisms are disproportionately stronger relative to their mass. This reasoning can be applied to any small life form, and not just to ants.

More questions on the same topic:

- \* Why are the best gymnasts young?
- \* Why is the ostrich flightless?
- \* Why are the elephant's legs so thick?
- \* Why do large bridges often bend in different places than their scaled-down models?

Why can insects not fly faster than about 20 km/h (and are usually much slower than even that)?



Short answer:

Because the air resistance is too strong.

Detailed answer with mathematical reasoning:

Air resistance depends on the object's cross-sectional area. In small animals, it is greater in proportion to their volume.

More questions on the same topic:

- \* Is it more efficient, per passenger and per kilometre, to fly in a large airplane or in a small airplane?
- \* When underwater, do smaller or larger bubbles of air rise faster to the surface?
- \* Why can crickets only jump about 50 cm in the air, even though they could, in theory, jump much higher?

Why do insect-sized animals not sink in water?



Short answer:

They are carried by the surface tension.

Detailed answer with mathematical reasoning:

In smaller animals, the surface area through which they touch the surface is greater in proportion to the animal's mass. For this reason, surface tension has a stronger relative effect on smaller objects.

More questions on the same topic:

- \* Why do flies and mosquitoes find it so easy to sit on walls?  
What about the gecko, and why would it be impossible for humans?  
Hint: Touching surface --> Force of adhesion + muscular strength --> no mass problem
- \* Which animal exerts the greatest pressure onto the ground? (And why is it not the elephant?)
- \* Can a needle made of gold swim on a liquid surface? If so, when or why?

Why do larger animals need a lung?

Short answer:

So that the blood stream can be steadily supplied with oxygen.

Detailed answer with mathematical reasoning:

As animals increase in size, their volume of blood rises by a power of 3.

The mixing of oxygen and blood can only happen on the surface, which grows by a power of 2 compared to the organism's size.

Thus, a strong increase in surface area has to happen somewhere:

In humans, the pulmonary alveoles cover a surface area of approximately  $100 \text{ m}^2$ .

More questions on the same topic:

- \* Why can't insects grow larger than they currently do?

